Introduction to 3-D Geometry

1. Locate the following points:
   - (i) $(1, -1, 3)$,
   - (ii) $(-1, 2, 4)$
   - (iii) $(-2, -4, -7)$
   - (iv) $(-4, 2, -5)$.

2. Name the octant in which each of the following points lies.
   - (i) $(1, 2, 3)$,
   - (ii) $(4, -2, 3)$,
   - (iii) $(4, -2, -5)$
   - (iv) $(4, 2, -5)$
   - (v) $(-4, 2, 5)$
   - (vi) $(-3, -1, 6)$
   - (vii) $(2, -4, -7)$
   - (viii) $(-4, 2, -5)$.

3. Let A, B, C be the feet of perpendiculars from a point P on the x, y, z-axis respectively. Find the coordinates of A, B and C in each of the following where the point P is :
   - (i) $A = (3, 4, 2)$
   - (ii) $(-5, 3, 7)$
   - (iii) $(4, -3, -5)$

4. Let A, B, C be the feet of perpendiculars from a point P on the xy, yz and zx-planes respectively. Find the coordinates of A, B, C in each of the following where the point P is
   - (i) $(3, 4, 5)$
   - (ii) $(-5, 3, 7)$
   - (iii) $(4, -3, -5)$.

5. How far apart are the points $(2, 0, 0)$ and $(-3, 0, 0)$?

6. Find the distance from the origin to $(6, 6, 7)$.

7. Show that if $x^2 + y^2 = 1$, then the point $(x, y, \sqrt{1-x^2-y^2})$ is at a distance 1 unit from the origin.

8. Show that the point A $(1, -1, 3)$, B $(2, -4, 5)$ and $(5, -13, 11)$ are collinear.

9. Three consecutive vertices of a parallelogram ABCD are A $(6, -2, 4)$, B $(2, 4, -8)$, C $(-2, 2, 4)$. Find the coordinates of the fourth vertex.

   [**Hint**: Diagonals of a parallelogram have the same mid-point.]

10. Show that the triangle ABC with vertices A $(0, 4, 1)$, B $(2, 3, -1)$ and C $(4, 5, 0)$ is right angled.

11. Find the third vertex of triangle whose centroid is origin and two vertices are $(2, 4, 6)$ and $(0, -2, -5)$.

12. Find the centroid of a triangle, the mid-point of whose sides are D $(1, 2, -3)$, E $(3, 0, 1)$ and F $(-1, 1, -4)$.

13. The mid-points of the sides of a triangle are $(5, 7, 11)$, $(0, 8, 5)$ and $(2, 3, -1)$. Find its vertices.

14. Three vertices of a Parallelogram ABCD are A $(1, 2, 3)$, B $(-1, -2, -1)$ and C $(2, 3, 2)$. Find the fourth vertex D.

15. Find the coordinate of the points which trisect the line segment joining the points

A (2, 1, – 3) and B (5, – 8, 3).

16. If the origin is the centriod of a triangle ABC having vertices A (a, 1, 3), B (– 2, b, – 5) and C (4, 7, c), find the values of a, b, c.

17. Let A (2, 2, – 3), B (5, 6, 9) and C (2, 7, 9) be the vertices of a triangle. The internal bisector of the angle A meets BC at the point D. Find the coordinates of D.

18. Show that the three points A (2, 3, 4), B (–1, 2, – 3) and C (– 4, 1, – 10) are collinear and find the ratio in which C divides AB.

19. The mid-point of the sides of a triangle are (1, 5, – 1), (0, 4, – 2) and (2, 3, 4). Find its vertices. Also find the centriod of the triangle.

20. Prove that the points (0, – 1, – 7), (2, 1, – 9) and (6, 5, – 13) are collinear. Find the ratio in which the first point divides the join of the other two.

21. What are the coordinates of the vertices of a cube whose edge is 2 units, one of whose vertices coincides with the origin and the three edges passing through the origin, coincides with the positive direction of the axes through the origin?

Objective Type Questions

Choose the correct answer from the given four options inidcated against each of the Exercises from 22 (M.C.Q.).

22. The distance of point P(3, 4, 5) from the yz-plane is

    (A) 3 units   (B) 4 units   (C) 5 units   (D) 550

23. What is the length of foot of perpendicular drawn from the point P (3, 4, 5) on y-axis

    (A) $\sqrt{41}$ (B) $\sqrt{34}$ (C) 5   (D) none of these

24. Distance of the point (3, 4, 5) from the origin (0, 0, 0) is

    (A) $\sqrt{50}$ (B) 3   (C) 4   (D) 5

25. If the distance between the points (a, 0, 1) and (0, 1, 2) is $\sqrt{27}$, then the value of a is

    (A) 5   (B) ± 5   (C) – 5   (D) none of these

26. x-axis is the intersection of two planes

(A) xy and xz   (B) yz and zx   (C) xy and yz   (D) none of these

27. Equation of y-axis is considered as

(A) $x = 0, y = 0$ (B) $y = 0, z = 0$ (C) $z = 0, x = 0$ (D) none of these

28. The point $(-2, -3, -4)$ lies in the

(A) First octant   (B) Seventh octant

(C) Second octant   (D) Eighth octant

29. A plane is parallel to yz-plane so it is perpendicular to :

(A) x-axis   (B) y-axis   (C) z-axis   (D) none of these

30. The locus of a point for which $y = 0, z = 0$ is

(A) equation of x-axis   (B) equation of y-axis

(C) equation at z-axis   (D) none of these

31. The locus of a point for which $x = 0$ is

(A) xy-plane   (B) yz-plane   (C) zx-plane   (D) none of these

32. If a parallelopiped is formed by planes drawn through the points (5, 8, 10) and (3, 6, 8) parallel to the coordinate planes, then the length of diagonal of the parallelopiped is

(A) $2\sqrt{3}$ (B) $3\sqrt{2}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

33. L is the foot of the perpendicular drawn from a point P (3, 4, 5) on the xy-plane. The coordinates of point L are

(A) (3, 0, 0)   (B) (0, 4, 5)   (C) (3, 0, 5)   (D) none of these

34. L is the foot of the perpendicular drawn from a point (3, 4, 5) on x-axis. The coordinates of L are

(A) (3, 0, 0)   (B) (0, 4, 0)   (C) (0, 0, 5)   (D) none of these